1. Introduction

1.1. What we are trying to do here

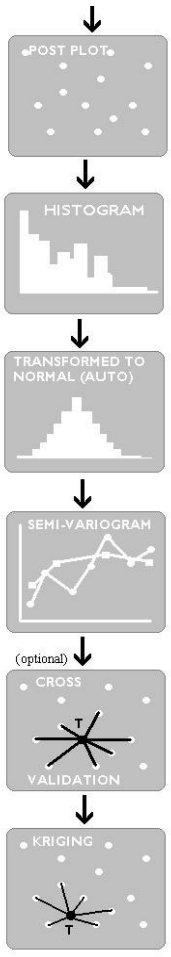
This volume is complementary to Practical Geostatistics 2000. It was produced as a further companion to *Practical Geostatistics 2000, Answers to the Exercises* on the suggestion of some of our teaching colleagues. They thought it would be a good idea to present the analysis of a data set as a"case study", following the analysis through from first statistical summaries to final kriging maps. Each chapter here is a study of a single data set.

To read this book and benefit from it you need to be fairly comfortable with basic algebra. That is, with the notion of using symbols as shorthand for longer statements. We have worked hard to bring you a consistent notation throughout the textbooks and keep the same conventions in these case studies.

The data sets and software used in these analyses can be downloaded from the Web at http://www.kriging.com/teaching_software.

As in the main textbook, we do advise you to keep your sense of humour and common sense to the fore at all times while reading this book. These sort of analyses work much better if you expect the answers to make sense.

1.2. Geostatistics in the real world



In many textbooks, documentation and professional reports, geostatistics is presented as a straightforward step-by-step process. Examples which are used for teaching processes are often chosen *because* they are straightforward and uncomplicated.

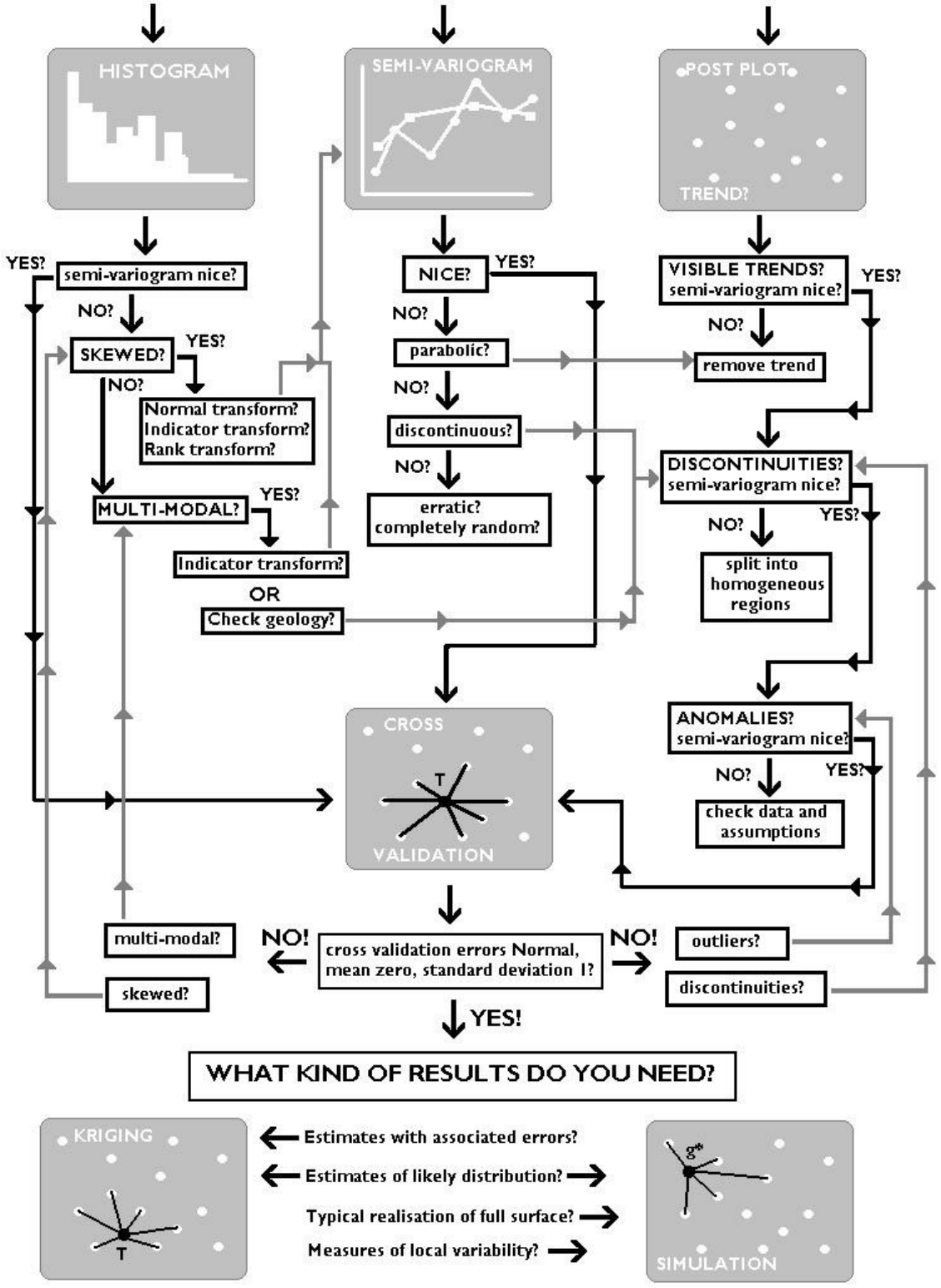
In this volume, we have tried to present examples which are a little more interesting than the usual classroom exercises. Most of these data sets are of reasonable size – from around 100 samples to over 5,000 samples.

Some analyses presented here follow the simple flow illustrated in the figure to the left. For example, the coal data is pretty linear.

Others are more realistic and require a more iterative approach – refining the interpretation and models to find an optimal technique for a particular data set. For example, the Velvet Leaf data set repays close attention to its statistical nature.

Some data sets, such as the scallops data, can be tackled in a variety of ways which may or may not produce similar results.

Finally, it is worth re-iterating that the case studies provided in this volume are intended as examples only and form no definitive guide as to how to analyse other data sets or similar problem.



Still better than the real thing

Comments and suggestions are always welcome from our readers – although, it has to be said, that positive suggestions are more gracefully received!

1.3. Notes for teachers

If you need an example and cannot find an appropriate one here, remember that:

- the behaviour of Normal data is not affected by adding a constant and/or multiplying by a factor;
- the behaviour of two parameter lognormal data is not affected by a multiplicative constant. Three parameter lognormals are more tricky.
- spatial relationships are not affected by a multiplying factor. If the data is Normal, with or without a trend, an additive constant will not affect trend analysis or other spatial relationships.

So, do what we do, use your imagination and experience to adapt these exercises for your own teaching purposes.

Precision: It is also important to remember that all of the answers given here have been computed using proprietary software, spreadsheets and/or hand calculators. All of these have different levels of precision in their makeup. Do not worry if your answer is out by anything up to a couple of percent compared to the one in this book. This is particularly so when you have to square, cube or raise numbers to a larger power.

If you make a real mistake, your answer will be quite different from ours. In most cases, mistakes during calculation lead to huge differences in the answers.

1.4. Cautionary note

Please note that the worked answers in this textbook are – especially in the later chapters – *possible* answers. It is a well established fact that different geostatisticians will produce different conclusions from exactly the same data set.

Both of us have experience of tackling applications where another (or many other) worker(s) have drawn their own conclusions. One of our biggest problems in the field of inference is that there are no *right* answers. There are *wrong* ones. There are *inappropriate* ones. There are *suboptimal* approaches. There are *silly* answers and there are *stupid* answers. There are even *funny* answers.

This volume contains *our* answers. Disagreement is your privilege.

1.5. Data sets

The applications presented within the book are geological (with a bias towards mineability) with some hydrology, soil science and environmental case studies. We have included applications of varied geostatistical techniques in order for the reader to see as many approaches in practice as possible in a volume of this size.

The *potential applications* of geostatistics include any form of measurable spatial data and some which cannot be given a quantitative measure, such as rock type, land use etc.

1.5.1. Coal:

a simulated set of data based on a real coal seam in Southern Africa. Boreholes drilled into the coal seam are measured for: thickness of coal (metres), energy content or 'calorific value' of coal (Megajoules per tonne); ash content (%) and sulphur content (%). Three co-ordinates in metres are available for the top of the coal seam where intersected by the drillhole.

In this example we study the following:

- relationships among the economic variables, including: covariances, correlations, predictive relationships between two variables and multi-variable relationships.
- a full study of the calorific value of the coal, including: summary statistics; histograms and probability plots; fitting a Normal distribution model; grade/tonnage curves from Normal model; inverse distance interpolation; calculation and modelling of semi-variograms; cross validation of the semi-variogram model; ordinary kriging.
- a similar study for the sulphur content of the coal samples.

1.5.2. GASA:

this data set is named for the Geostatistical Association of South Africa and was used in an illustration of geostatistical techniques at a meeting in April 1987 in Johannesburg. The sample data are taken from deep boreholes drilled into a typical Witwatersrand type gold reef. The measurements of interest are the grade of the gold in grams per tonne of rock (parts per million) and the thickness of the reef intersection in the borehole (centimetres). The 27 boreholes lie approximately 1 kilometre apart and constitute a typical data set for the planning and design of a new Wits gold mine. The values have been disguised by a factor but are otherwise unaltered. Co-ordinates are in metres.

This case study is all about dealing with highly skewed data using the lognormal distribution. We investigate simple variations on the lognormal, namely the "three parameter" lognormal model. In this model, a constant is added to each sample value to provide data which will fit the standard lognormal.

We discuss:

- fitting the distribution models;
- estimation of the mean and standard deviation;
- confidence intervals for both logarithmic and backtransformed parameters;
- grade/tonnage curves using these models.

For this case study, we have been able to obtain some underground development data at a spacing of around 1.4 metres. This enables us to get a generic semi-variogram which we can use for theoretical calculations of grade/tonnage curves for this project.

We also discuss briefly the possibility of using a single economic variable – the accumulation in centimetre grams per tonne.

1.5.3. Geevor:

Mining in Cornwall dates back to between 1000 and 2000 B.C. when Cornwall is thought to have been visited by metal traders from the eastern Mediterranean. They even named Britain as the "Cassiterides' - "Tin Islands'. Cornwall along with the far west of Devon provided the vast majority of the United Kingdom's tin and arsenic and most of its copper. Initially the tin was found as alluvial deposits in the gravels of stream beds, but before long some sort of underground working took place. In fact, where the tin lodes outcropped on the cliffs underground mines sprung up as early as the 16th century.

This study concerns sample data from a hydrothermal tin vein deposit in Cornwall, England. From the Online version of the Encyclopedia Britannica:

> **Main article: vein** in geology, ore body that is disseminated within definite boundaries in unwanted rock or minerals (gangue). The term, as used by geologists, is nearly synonymous with the term lode, as used by miners. There are two distinct types: fissure veins and ladder veins.
>
> **hydrothermal ore deposit** ..... a vein, which forms when a hydrothermal solution flows through an open fissure and deposits its dissolved load.

The mineralisation appears as a continuous vein which is almost vertical. Samples of around 1kg are chipped across the vein, which averages about 24 inches wide. Measurements are grade of tin in pounds of black tin (SnO_2) per ton of rock. The thickness of the vein or 'lode' is measured to the nearest inch. Co-ordinates are in feet along section and elevation above an arbitrary base level.

Clark, I., 1979, "Does geostatistics work?", Proc. 16th APCOM, Thomas J O'Neil, Ed., Society of Mining Engineers of AIME Inc, New York, 213-225.

Clark, I., "Geostatistical Reserve Estimation in Cornish Tin Mining", Ph.D. Thesis, University of London, 1979

In this case study, we discuss:

- the statistical behaviour of the vein width and the tin value and the complexities introduced by the deposition of the mineral(s).
- whether or not to combine these two variables into a single accumulation value for economic analysis.
- semi-variogram calculation and modelling for both variables, using development samples only.
- whether the nugget effect can be interpreted as sampling error or inherent geological variability (or both!). The assaying technique which is used to produce the measured grades is examined in detail with a special sampling scheme.
- cross validation of the semi-variogram for grades, using stope samples to check back on a model calculated on development samples.
- comparing lognormal kriging with the lognormal shortcut – which kriges on untransformed values.

The interesting conclusion from this work is that lognormal kriging can be used in this case, even though the underlying statistical distribution is not lognormal. A side issue is also resolved when we find that stope samples can be cross validated from a model produced from development drives.

1.5.4. Organics:

Data are Soil Organic Matter values (in grams per kilogram) derived from soil samples taken in a research field at the University of Nebraska West Central Research and Extension Center near North Platte, Nebraska, USA. Data were taken as part of experiments on variable-rate fertilizer technology. Co-ordinates are in metres.

- *Reference: Gotway, C.A. and Hergert, G.W. (1997). "Incorporating Spatial Trends and Anisotropy in Geostatistical Mapping of Soil Properties". Soil Science of America Journal, 61:298-309*

The abstract for the cited paper reads:

> The spatial variation in soil parameters often differs with direction. These differences may occur naturally or may be due to management practices. Regardless of their origin, they present a challenge in geostatistical mapping of soil parameters. Recommendations pertaining to the selection of an appropriate geostatistical method based on the current literature are often incomplete or contradictory. The purpose of this investigation was to provide a unified description, comparison, and discussion of different geostatistical methods for handling trend and anisotropy that may be present in measured soil properties. Soil organic matter content of the 0- to 20-cm depth from a field in continuous ridge-tilled corn (Zea mays L.) was used to compare five geostatistical methods: ordinary kriging with an isotropic semi-variogram (OKI); ordinary kriging with an anisotropic semivariogram (OKA); ordinary kriging within local neighborhoods (OKN); universal kriging (UK); and median polish kriging (MPK). Organic matter maps produced from the five methods showed similar large-scale features but marked differences in the finer features. A comparison of percentage of total area in each organic matter range among mapping methods also showed strong similarities; however, the proportion of the field assigned to each range differed by as much as 7%. Larger differences would be expected at large sample spacing. Although the five methods produced similar maps, selection of the "best" technique should be based on selection of an associated model that best accounts for and describes the nature of the cause of the variation.

Here we present a slightly different approach to the data.

- Statistical analysis reveals a possible outlier with a very low value compared to all other sampling. We discuss the problem briefly and continue to analyse the data with this in mind.
- Inverse distance is used to sketch maps and to illustrate the question of the appropriate distance function to use for estimation purposes. We see that there appears to be a strong anisotropy in the maps.

- The semi-variogram is calculated. Our interpretation of the semi-variogram is that there seems to be a strong trend in the values.
- Trend surface analysis is carried out and the significance of the trend tested in the usual ways (cf. Chapter 7 in Practical Geostatistics 2000.
- The semi-variogram is recalculated using residuals from the fitted cubic trend surface. This semi-variogram can be modelled.
- Cross validation is undertaken and we find that there are now two significant outliers in the data: one is the extreme low value found previously; the other is an apparently acceptable sample value which just does not mesh with the neighbouring sample values around it.
- Universal kriging is carried out for the full data set and for the data set less the two apparent outliers. Comparisons are illustrated and discussed.

It is interesting to note that, while Universal Kriging was one of the 5 methods used in the original paper there was no discussion of any outliers in the data or of what to do with them.

1.5.5. Scallops:

Scallop data were collected during a 1990 survey cruise off the east coast of North America. Scallop counts were obtained using a dredge. Any scallop smaller than 70 mm was termed a prerecruit. Total catch is the sum of prerecruits and recruits. Measurements included in the data file are:

- National Marine Fisheries Service (NMFS) 4 digit strata designator in which the sample was taken;
- sample number per year ranging from 1 to approximately 450;
- location in terms of latitude and longitude of each sample in the Atlantic Ocean;
- total number of scallops caught at the sample location;
- number of scallops whose shell length is smaller than 70 millimeters;
- number of scallops whose shell length is 70 millimeters or larger.

Reference: Ecker, M.D., and Heltshe, J.F. 1994. "Geostatistical estimates of Scallop Abundance", In, Case Studies in Biometry, Lange et al., editors. Wiley, New York

This is a marvellous data set, provided by colleague Carol Gotway Crawford, now of CDC in Atlanta. This data set can be analysed in more ways than "you can shake a stick at". In this section, you will see:

- summary statistics for "total catch" with discussion of precision problems introduced by large sample values;

- discussion of Normal distribution assumption with calculation of confidence intervals and selectivity abundance/catch limit calculations;
- discussion of a possible lognormal model, fitting of an additive constant and comparison of abundance/catch limit from theoretical model and actual data;
- inverse distance interpolation, a worked example and a full map with discussion of possible anisotropy;
- lognormal geostatistics: semi-variogram calculation, cross validation, backtransforms and lognormal kriging;
- rank uniform transforms: a distribution-free approach to geostatistics to find patterns of abundance.

You could also try – for your own benefit – lognormal geostatistics with trend: with full trend surface analysis on logarithmic values and universal lognormal kriging; Alternatively you could try a two stage process, with indicator kriging for presence/absence of scallops and lognormal kriging for when scallops are present (similar to Velvet Leaf example).

1.5.6. Sunshine Mine

This data set was provided by Pierre Mousset-Jones of Mackay School of Mines in Reno, Nevada. The data is a historical sampling set from development drives in an almost vertical vein. Silver and gold assay are available, as well as the width of the vein.

This study illustrates:

- fitting of lognormal distributions with additive constant;
- estimation of mean value and confidence levels, using large sample theory;
- grade tonnage curves for three parameter lognormal distribution;
- calculation and modelling of logarithmic semi-variograms;
- kriging maps compared with inverse distance squared;
- estimation of the average value over mining panels.

1.5.7. Velvetleaf:

Subsample of the number of velvetleaf weeds counted in 7 meter^2 area in a field in Nebraska. Data were collected by Gregg Johnson (see 2nd reference), as part of a research program in weed management at the University of Nebraska.

References: Data set taken from: *Gotway, C.A., and Stroup, W.W. 1997. "A generalized linear model approach to spatial data analysis and prediction". Journal of Agricultural, Biological, and Environmental Statistics, 2:157-178.*

Data collected by: *Johnsen, G.A., Mortensen, D.A., and Gotway, C.A. 1996. "Spatial and temporal analysis of weed seedling populations using geostatistics". Weed Science, 44:704-710.*

This case study illustrates the need for a statistical analysis of the data before venturing into a geostatistical analysis. This chapter includes sections where:

- We illustrate the sensitivity of inverse distance interpolation to choice of distance function and search radius;
- calculation and modelling of semi-variogram graphs;
- cross validation of the semi-variogram model.

We find that the cross validation exercise is extremely unsatisfactory – even though the average and standard deviation of the error statistics are close to the ideal [0,1]. We start our analysis again by first considering the statistical behaviour of the values. We find that the data is highly positively skewed but not lognormal (or any variation thereof). Studying a logarithmic probability plot leads us to the conclusion that there are actually three "populations" within the study area:

- quadrats with no weeds (absence of weeds);
- quadrats with up to 40 weeds (colonisation?);
- quadrats with more than 40 weeds (climax vegetation?).

We illustrate the usefulness of indicator transforms for the characterisation and location of the various stages of weed growth, including indicator semi-variogram and kriging.

It becomes apparent from this map that the field under study is largely non-homogeneous and that further investigation into the previous history of the area might provide more realistic estimation techniques.

1.5.8. Wolfcamp:

Measurements of water pressure (potentiometric level) in 85 water wells in the Texas panhandle. This data set was part of a study carried out by the Office for Nuclear Waste Isolation in the mid 1980s on a potential site for a high level nuclear waste repository. The Wolfcamp aquifer underlies the planned repository. One aspect of repository planning is to quantify the risks inherent in a breach of the storage facility. Should radionuclides leak into the local aquifers, the scope and speed of potential contamination has to be assessed. The pressure of fluid within the aquifer was one of several variables used to determine the travel path and speed of travel for escaped radionuclides.

Reference: Harper, W.V., and Furr, J.M., 1986. "Geostatistical analysis of potentiometric data in the Wolfcamp Aquifer of the Palo Duro Basin, Texas", BMI/ONWI-587, April, Office of Nuclear Waste Isolation, Battelle Memorial institute, Columbus, Ohio.

This study of the Wolfcamp data illustrates:

1. descriptive statistics, the Normal distribution and its applicability to this data;
2. nearest neighbour analysis and sketch maps produced by inverse distance methods;
3. semi-variogram calculation, identification of extremely strong trend in the values;

4. trend surface analysis, recalculation of the semi-variogram on residuals from the trend;
5. modelling of the residual semi-variogram and cross validation;
6. kriging 'point' maps and polygonal averages.

1.5.9. Brooms Barn

This is another soil science application, data supplied by Dick Webster – co-author of Webster & Oliver's excellent "Geostatistics for Environmental Scientists". Brooms Barn is an agricultural experimental station in East Anglia (UK) which hosts several fields within its area. The data set includes Potassium (K mg/l), Phosphorus (P mg/l) and pH levels in the soil. Over 400 samples were collected on a regular grid at 40 metres spacing. The data file consists of 434 samples and the following fields for each sample:

- East and North position on the sampling grid – this is not in metres but in grid spacing, i.e. 1 unit of distance equals 40 metres;
- K – potassium value in the soil, mg/l;
- log10 K – logarithms to the base 10 of K values;
- pH – universal measurements for acidity (or lack of) in the soil;
- P – phosphorus levels in the soil, mg/l;
- log10 P – logarithm to the base 10 for P values.

It is obvious from the data file that Dr Webster had identified positive skewness on the K and P values. In our analysis of this data, we use the K values for illustrative purposes. We show that the K values are moderately skewed but that untransformed semi-variograms are adequate once a certain number of apparent outliers have been identified. Thanks to Google Earth^{TM} we can see where these outliers are located and posit reasons for eliminating them from the data set.

We also present a brief analysis of the pH (acidity) values for contrast with the K analysis. This variable shows inhomogeneities which are interpreted as two "populations" within the study area. We show how an indicator transform leads us to the conclusion that three fields have probably been treated with lime or some other 'basic' preparation. We show how pH could be mapped if the field boundaries are not known, using a combination of indicator and ordinary kriging. We compare this compound analysis with a map kriged using the indicator semi-variogram to approximate the actual pH semi-variogram.

1.5.10. Brenda Mine

Our final example in this volume was provided by our esteemed colleague Pierre Mousset-Jones. The major feature of this data set is that it is a full three dimensional example. Brenda Mine is a porphyry copper/molybdenum deposit in the southern interior of British Columbia, approximately 22 kilometers west of Peachland in the Central Okanagan, and was closed for economic reasons around 1990. In its time Brenda Mines processed 182 million tonnes of rock resulting in:

Metals Produced

Copper	278,000 tonnes
Molybdenum	66,000 tonnes
Silver	125 tonnes
Gold	2 tonnes

The files provided by Professor Mousset-Jones were in standard mine planning format, with separate files for borehole collar co-ordinates, survey information and mineral values. These were combined into a single text data file for teaching purposes. The data file `Old_Brenda.dat` contains 1,856 samples. Core sections are between 2 and 56 in length (we assume this to be feet). The longer cores are generally very low grade The data file supplied at http://www.kriging.com comprises the following information:

- X_co-ordinate
- Y_co-ordinate
- Z_co-ordinate
- Cu%
- Mo%
- length_of_core
- From (top of core section sample relative to collar)
- To (bottom of sample relative to borehole collar)

The analysis described here is a fairly straightforward one with comments on traditional geological sampling along section lines.

The Copper (Cu) values are used for our illustration. A very few extreme outliers are identified. Three dimensional semi-variogram graphs are constructed and discussed.

Cross validation is used to determine the necessary order of trend to be used for kriging purposes. Possible outlier values are discussed.

Kriging is applied to produce example maps and accompanying standard errors.

Block kriging is discussed briefly and a small example shown.

Finally, we mention why we did not use trend surface analysis.