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Direct Geostatistical Estimation of Irregular 3D Volumes

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Those who have had occasion to apply geostatistics to the estimation of ore reserves are familiar with the conventional approaches of estimating average grade or quality values for arrays of regular grid squares or blocks. Further, the technology to perform geostatistical analysis and prediction for irregular two-dimensional areas has been available in a practical, readily useable form for some time. However, the generalization of this technology into three dimensions has been hampered by lack of a suitable geometrical characterization for realistic, three-dimensional shapes and volumes.

The extension of these estimation techniques into three dimensions has now been made possible by a successful integration of the volume modelling capabilities of 3D Component Modelling with proven 3D Geostatistical Techniques. The result is a powerful new technique for mineral ore reserve estimation in a practical, useable form. The technique allows direct geostatistical estimation of grade and quality values for precisely defined, irregular, realistic, geological and mining shapes and volumes.

This paper describes the two technologies, their integration and their application in practice to the estimation of mineral ore reserves. The paper closes with a discussion of the benefits provided by the new approach, in terms of accuracy, efficiency and practicality.

DIRECT GEOSTATISTICAL ESTIMATION OF IRREGULAR 3D VOLUMES

INTRODUCTION

Mineral ore deposits exist in three-dimensional geological space in varying degrees of complexity. For detailed evaluation and mine-planning purposes we need to be able to represent deposits, and simulate their extraction, within a computer environment. For successful and efficient representation the modelling process must be able to accommodate ALL aspects of three-dimensionality to a level of precision which is compatible with an acceptable degree of mining and financial risk. Traditional modelling methods are generally incapable of fulfilling this requirement.

This paper describes a new modelling technology which satisfies the precision criteria in all aspects. The technology is based on the successful integration of new 3D Geostatistical theory, which allows the direct estimation of irregular three-dimensional volumes, and 3D Component Modelling, which provides the necessary precision in geometrical representation of three-dimensional shapes and volumes.

BACKGROUND

A few deposits, like thin uniform tabular coal seams, can be approximated by gridded surface representation. A few others, like simple disseminated base metal deposits, can be approximated by traditional block modelling techniques. The majority of deposits exhibit a degree of complexity which renders their representation by traditional modelling methods unacceptable. The complexity may be due to structure (faulting, folding, intrusions, etc.), irregularity in shape and form, variation in grade or quality, or combinations of these and other three-dimensional effects. In all cases the complexity is increased by an order of magnitude when the need to represent three-dimensional excavation limits, and to simulate the extraction of the deposit, is superimposed on the modelling requirement.

The traditional computer modelling methods all involve approximations which introduce errors when applied to complex three-dimensional ore deposits. These errors affect the end results of any deposit evaluation or mine-planning exercise. Even less acceptable, most traditional methods are incapable of providing any measure of the magnitude of the estimation errors introduced. To elaborate, any geostatistical estimate of grade or quality is incomplete (and misleading) unless it provides not only an *average* value, but also a *range* and an associated *confidence level* (or probability) for the volume of concern.

Thus the reliability of an estimate produced by traditional modelling methods is unknown and cannot be related to an acceptable degree of risk. Considering the amount of capital required to implement a mining project, the high cost of operation, and the associated risks of today's volatile commodity markets, the elimination of these deficiencies by the new technology presented below is indeed timely.

Three essential features are required in order to successfully model complex three-dimensional deposits, and to simulate their extraction, within a computer.

- An ability to define and represent irregular, realistic, geological and mining shapes with precision.
- A precise method of determining the volumes of these irregular three-dimensional shapes, and their volumes of intersection.
- A method for direct geostatistical estimation of grade or quality within these irregular three-dimensional volumes which accommodates the geometry of geological and mining shapes.

The first two requirements have been met in THEORY AND PRACTICE by the recently developed and proven capabilities of 3D COMPONENT MODELLING technology, as conceived by LYNX Geosystems Inc. and implemented within the LYNX Mining System.

The third requirement has previously been met IN THEORY ONLY by 3D GEOSTATISTICS technology, as conceived by Geostokos Ltd.. The implementation of this theory in practice has been hampered by lack of a suitable geometric characterization for irregular three-dimensional volumes with which it can be integrated.

This last requirement has now been provided in a compatible and complementary form by 3D Component Modelling.

The product of combining and integrating these technologies is a computer modelling methodology which provides a practical, logical and precise approach to estimation and evaluation of complex ore deposits. It also provides a stable platform on which to base mine-planning and production scheduling activities. The methodology eliminates the use of traditional modelling methods and their inherent approximations, constraints and errors, and considerably reduces the amount of computer storage and processing required. Finally, it provides a measure of reliability (associated standard error) for any estimate produced. The results of its application can therefore be related directly to an acceptable degree of mining and financial risk.

The two technologies, their successful integration, and their application in practice to the evaluation and mine-planning of mineral ore deposits are described below.

THEORETICAL BASIS

The theory, process and application of 3D Component Modelling technology have been described in detail elsewhere (1,2). The key features which distinguish this technology from traditional modelling methods are the following.

- The ease and precision with which complex irregular shapes are defined and represented in the computerized geological environment. This is achieved by means of a geometric characterization of shapes termed 3D Solids Modelling; based on analytical geometry and solids of integration theory.
- The precision with which the enclosed volumes of complex irregular shapes, and the volumes of intersection of two or more shapes, are determined. This is achieved by means of an analytical procedure termed 3D Volumetrics; based on the geometry of intersection of any plane with an irregular shape, and volumetric integration.
- The precision with which spatially variable characteristics, such as grade and quality, associated with the volumes of irregular complex shapes are estimated and represented. This function was provided in the previous implementation of 3D Component Modelling by an integration of accepted geostatistical procedures with the features described above. This development in itself represented a significant advance in modelling technology.

In the present implementation the accepted procedures are replaced by the 3D Geostatistics technology described herein. This technology is in all ways more compatible and complementary to the geometrical characterization of shapes used by 3D Solids Modelling, and to the analytical procedures of 3D Volumetrics. The technology is an extension of existing geostatistical methods for the estimation of irregular two-dimensional polygonal areas into the third dimension. Its implementation is based on the ability to readily determine the polygonal geometry of intersection of planes with an irregular shape. The basic requirements are therefore similar to those of 3D Volumetrics.

Within 3D Component Modelling, the complex irregular shapes of realistic geological and mining units are modelled by sets of one or more 3D Solids components, as described below. It is the unique way in which these components are defined and represented within the computer, and the ability to determine the geometry of intersection of a plane (or section) at any orientation, which make possible the integration of the 3D Geostatistics technology. In order to present a meaningful description of the new technology and its integration, a brief preliminary description of these features is required.

1. SOLIDS MODELLING OF IRREGULAR 3D SHAPES

A typical 3D Solids component, representing an irregular three-dimensional shape (or part of a shape), is illustrated by figure 1.

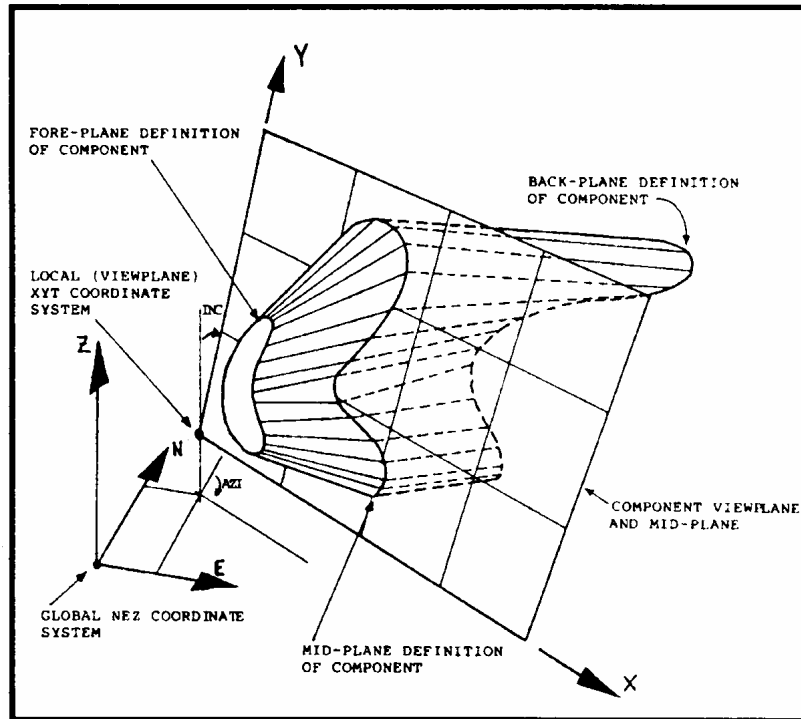


Figure 1: The definition and geometric characterization of a 3D Solids Modelling Component representing an irregular volume

The orientation of the component is controlled by that of the viewplane from which it is defined. This is specified in terms of N,E,Z,Azimuth and Inclination values in the Global Coordinate System. The shape of the component is controlled by Mid-plane, Fore-plane and Back-plane boundaries, defined interactively, and by the links between boundary points. The Fore-plane and Back-plane are assumed parallel to the Mid-plane and are therefore fully defined by a specified Fore-thickness and Back-thickness respectively. The points comprising the boundaries are defined in terms of Local (XYT) System coordinates. The volume of the component is that enclosed by the implied polygonal facets formed by the boundaries and links of the component. This completes the geometrical characterization of the component.

The component definition is completed by specification of several additional items, including unit and component identification strings. These are specified according to a selected convention and are referenced by the wild-card retrieval facilities of the modelling database. In this way *sets* of contiguous components can be used to represent more complex geological and mining units.

Despite the simplicity of the geometrical characterization of components, it has been successfully and efficiently applied to the representation of all forms of complex geological and mining shapes. Figure 8 at the end of the paper illustrates these capabilities.

2. SECTIONS AT ARBITRARY ORIENTATIONS

A feature of 3D Solids Modelling, based on three-dimensional analytical geometry and the geometric characterization of components, is the ability to readily determine the two-dimensional polygonal geometry of intersection of a plane with a component, irrespective of their relative orientation. This capability is illustrated by figure 2. It allows the display of sections at any orientation through irregular three-dimensional shapes represented by sets of one or more contiguous components. This capability also provides the basis for volumetric integration and 3D geostatistical estimation, as described below.

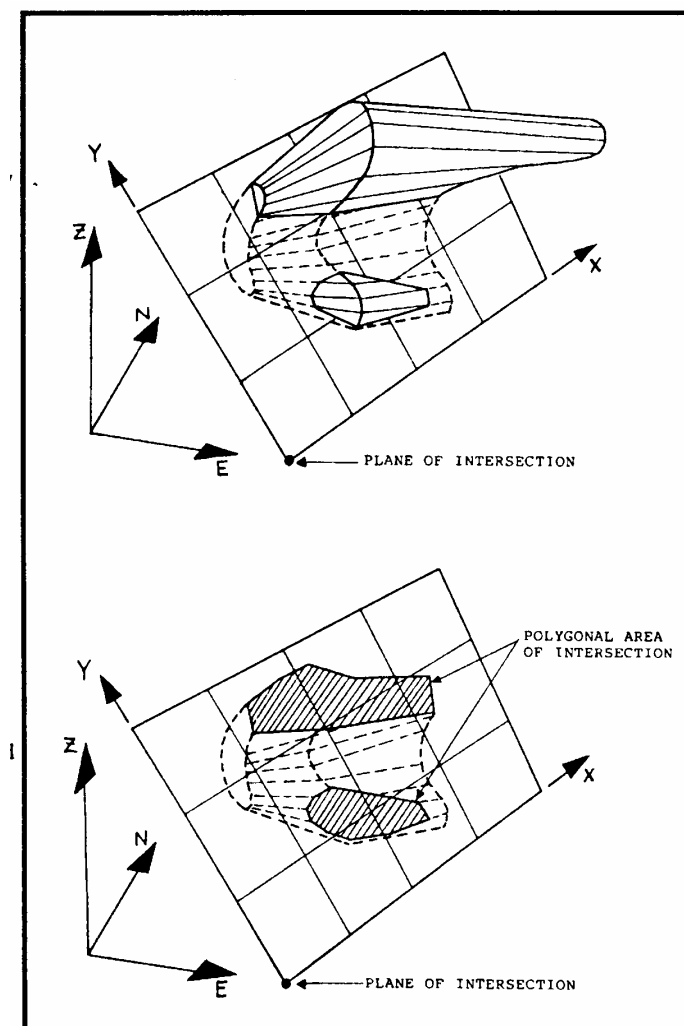


Figure 2: Top: Component intersected by arbitrary plane (refer figure 1)
Bottom: Polygonal geometry of intersection

3. VOLUMETRIC INTEGRATION OF IRREGULAR SHAPES

The process of determining the two-dimensional geometry of intersection of a plane with a component can readily be extended to determination of the area of intersection by means of simple analytical geometry. If the concept of a thickness, measured normal to the plane, is added then the result is an intersection *slice* with an associated volume. If the plane of intersection is replaced by a set of parallel planes at a constant spacing, then a volume of intersection can be obtained for each of the planes. Integration (accumulation) of these volumes throughout a component provides a precise measure of the volume of the component, providing that the spacing, or integration increment, is sufficiently small. The process is graphically illustrated by figure 3.

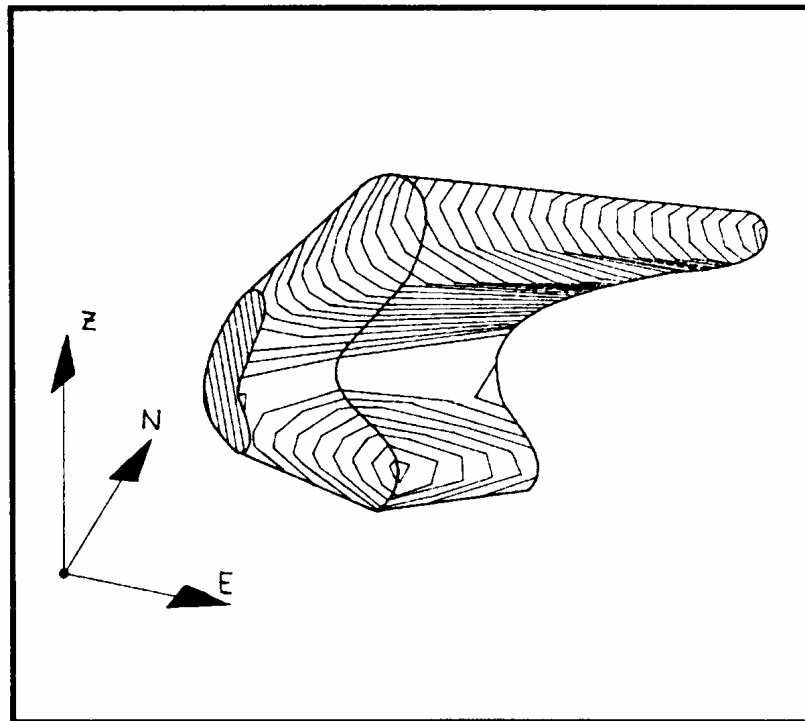


Figure 3: Graphical representation of parallel intersection planes through a component, used for volumetric integration (refer figure 2)

The volumetric integration process is extended from single components to the analysis of complex irregular shapes by accumulation of results for the complete set of components representing the shape. The process is independent of the orientation of the slices relative to the components. Thus there is no requirement for all of the components representing a geological or mining unit to be similarly oriented.

4. GEOSTATISTICAL ESTIMATION OF IRREGULAR VOLUMES

The primary objective of the 3D Geostatistics technology is to produce directly a single estimate, and its associated standard error, for irregular realistic geological and mining volumes. The requirement for this capability is based largely on the following.

The estimation of the average value over a volume differs in concept, theory and practice from the estimation at a single point. It is a fact that the average value of all points within a mining slope, or a panel, has a different distribution of values from that of the original sample data. In particular, average values tend to vary less than sample values. Thus the variance of slopes or panels tends to be lower than that of relatively small samples. In a similar way, the relationship between a sample and a large volume is not the same as between the sample and (for example) the centre of mass of that volume. Kriging is possibly the only estimating technique which incorporates the geometry of the *volume to be estimated* into the estimation process.

For this reason, the representation of geological and mining volumes by multitudes of blocks and sub-blocks (as in traditional block modelling) not only introduces errors of approximation at the volume boundaries but also introduces errors of estimation throughout the volume. In the latter case the estimation error increases as the blocks are made smaller. This situation is compounded by an inability to produce any useful measure of the estimation error, caused by sample distribution and volume geometry. All of these approximations and deficiencies are eliminated if irregular volumes are treated as *whole* units, as presented below.

Since the new technology is an extension of the existing approach for estimation of two-dimensional irregular areas, it is instructive to summarize the latter first (3).

The major difference in technique between kriging the value at a point and obtaining the average value for an area (or volume) is in the setting up of the system of simultaneous equations which yield the kriging estimator and its associated standard error. These equations are dependent on the size and shape of the area and its relationship to the known samples. This factor becomes obvious on inspection of the kriging estimation equations, and the variance equation, in their generic form (6), as follows.

The kriging estimation equations

$$\begin{aligned}\sum w_i \bar{\gamma}(s_i, s_j) + \lambda &= \bar{\gamma}(s_i, v) \quad [\text{for each sample}] \\ \sum w_i &= 1\end{aligned}$$

and the kriging variance equation,

$$\sigma^2_K = \sum w_i \bar{\gamma}(s_i, v) + \lambda - \bar{\gamma}(v, v)$$

where,

w_i is the sample weighting factor
 s_i is the sample value
 $\bar{\gamma}(,)$ is the semi-variogram function
 v is the volume (area) of concern
 λ is the associated standard error

In order to set up these equations the following semi-variograms must be calculated and their contributions to the equation coefficients determined. g_i

- (i) Between each pair of samples; represented by the term $\bar{\gamma}(s_i, s_j)$ in the equations.
- (ii) Between each sample and every point within the irregular area; represented by the term $\bar{\gamma}(s_i, V)$.
- (iii) Between every possible pair of points within the irregular area; represented by the term $\bar{\gamma}(V, V)$.

In (ii) above, instead of the semi-variogram between two points, ie. a sample and the point being estimated, the requirement is to calculate the average semi-variogram between the sample and EVERY point in the specified area.

The necessary contribution to the kriging equation corresponding to the sample can then be made. This process is repeated for each available sample.

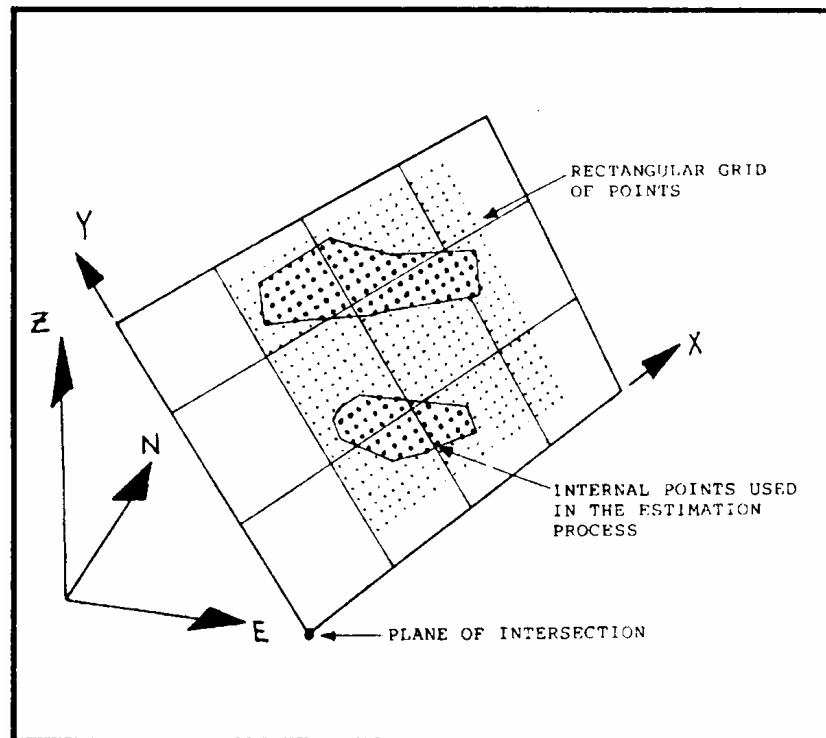


Figure 4: The rectangular grid of discrete internal points used in the estimation of a two-dimensional polygonal area (refer figure 2)

To perform this process in the case of an irregular area (or polygon) a numerical approximation is used. This involves placing a rectangular grid over the polygon and selecting every point which falls inside the polygon. These discretized points are assumed to represent the whole area. Much investigation has been done on determining the number of grid points required to obtain a suitable level of precision (4),(5). It has been found that 100 (approximately) points falling inside a polygon provides an acceptable result for most shapes; the grid density can be adjusted to suit the size and shape (or degree of irregularity). It should be emphasized that this discretization is used only to represent the contribution of the area itself to the kriging equations. Figure 4 illustrates the discretization concept.

The final quantity of concern is the estimation (or kriging) variance. This is partially dependent on the variability of values within the area being estimated, and therefore is also sensitive to the size and shape of the area. The average variability within the area is obtained by calculating the semi-variogram value between every possible pair of points within the area and taking the average value (refer (iii) above). The necessary contribution to the kriging variance equation can then be made. The same numerical approximation grid as above is used in this case.

Solution of the set of kriging equations resulting from this process yields the sample weighting factors, and hence a single estimated (kriged) value, and the associated standard error for the irregular area.

The extension of the estimation process into three dimensions follows the same steps as those outlined above for the two-dimensional process. In fact the only real difference is in the mechanics of the discretization process itself.

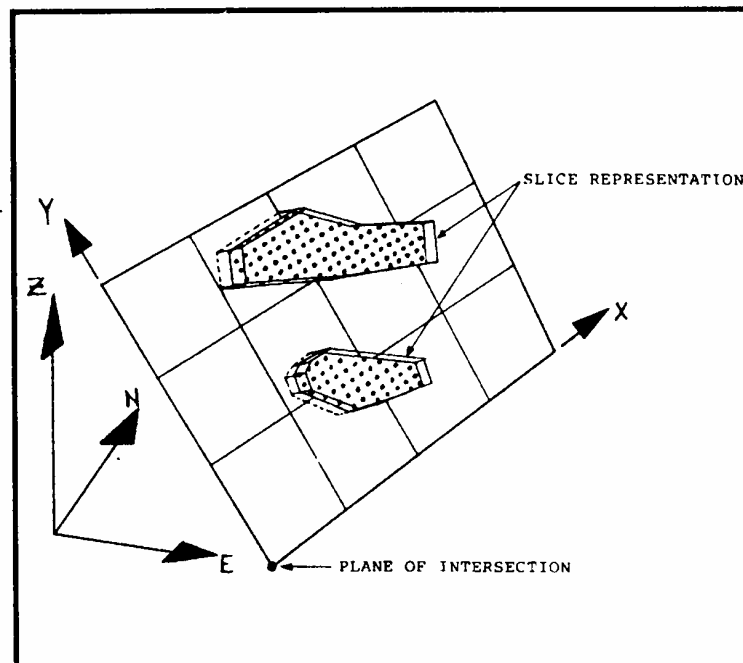


Figure 5: The transition to three-dimensions – an equivalent “slice” and its representative discretization and geometry (refer figure 4)

Assume, as in the case of volumetric integration of irregular shapes (refer 3. above), that the polygonal boundary of the area of intersection of a plane with a component is representative of the geometry of a slice of uniform thickness. Assume further that the discretization of the two-dimensional area by a uniform grid of points is equally representative of the volume of the slice. If the plane of intersection is replaced by a set of parallel planes at a constant spacing, equal to the *slice* thickness, and the two-dimensional discretization process is repeated for each plane of intersection, then the result is a three-dimensional discretization of the volume of the component. The spacing of the grid points in the third dimension is equal to the slice thickness and the set of contained points is representative of the volume. Figure 5 illustrates the slice concept.

The slice thickness in this case is independent of the thickness used for volumetric integration purposes and depends solely on obtaining a sufficient number of internal points to provide adequate discretization of the volume.

In this implementation of the technology the grid density and the slice thickness are variables which can be adjusted to suit this requirement.

The semi-variogram values between samples and internal points are accumulated for the entire volume during the slice discretization

process, averaged at the end, and contributed to the kriging estimation equations. The semi-variogram values between all possible pairs of internal points are, of necessity, only determined and accumulated once discretization of all slices is complete, ie. once the final form of the representative three-dimensional set of internal grid points is known. The average variability within the volume is then determined and contributed to the kriging variance equation.

As before, solution of the equations yields a single estimated kriged value for the irregular volume, and its associated standard error.

Orientation (of planes, slices and grid), grid spacings and slice thickness are all variables which can be adjusted to suit the characteristics (geometry, anisotropy, etc.) of the problem at hand. In the case of a more complex irregular shape, the mechanics of the three-dimensional discretization process are readily extended to include the concept of representation of the shape by a set of contiguous components.

5. EXTENSION TO VOLUMES OF INTERSECTION

The volumetrics and geostatistics procedures described above (refer 3. and 4.) are readily extended to handle volumes of intersection between complex irregular shapes. Consider volumetrics first and the case of two independent components which intersect each other (refer figure 6).

Select a plane which intersects both of the components, each of which produces a polygonal area of intersection on the plane (refer figure 6). The area of intersection common to both components is readily determined by geometrical analysis of the intersection of polygons. The result is a new set of *intersection* polygons. As before, application of the *slice* concept, replacement of the intersection plane by a set of parallel planes, and accumulation

of the slice volumes provides a precise measure of the volume of intersection of the two components. A graphical presentation of this process is provided by figure 7.

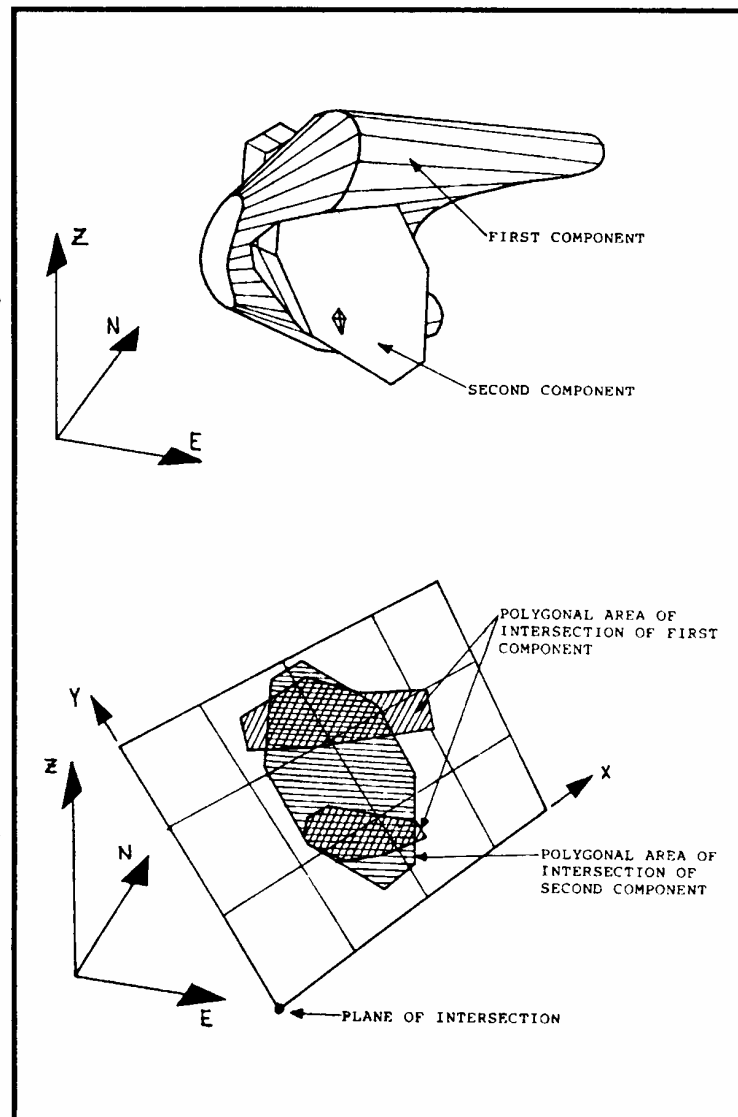


Figure 6: Top: Two components which intersect each other
Bottom: A common intersection plane and the overlaid polygonal areas of intersection

Similarly, application of this intersection slice technique to the geostatistics procedures outlined above (refer 4.) leads to a direct kriged estimate, and its associated standard error, for the volume of intersection of any two irregular shapes.

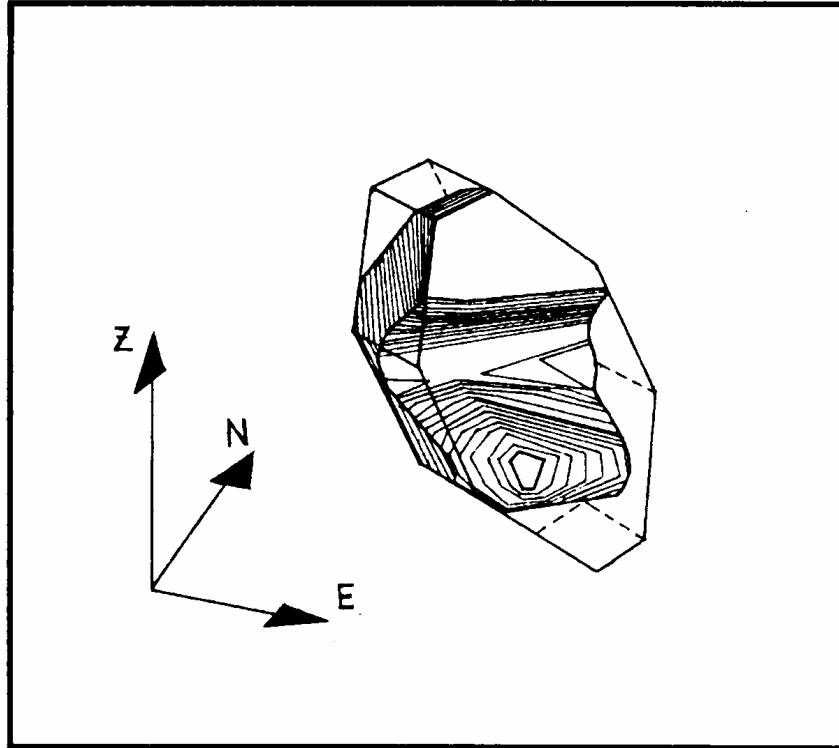


Figure 7: Graphical representation of parallel intersection planes through the intersection of two components, used for volumetric integration (refer figure 6)

TECHNICAL ADVANTAGES

The obvious technical advantages of the 3D Geostatistics technology and its integration with 3D Component Modelling are many, these are listed below (there are no perceived disadvantages at this stage).

- The boundary approximations inherent to most traditional modelling methods are eliminated in the estimation process.
- The size and shape of irregular volumes, and their relationship to associated samples, are taken into account in the estimation process; ie. the form of the resulting kriging equations is geostatistically correct.
- The methodology provides direct geostatistical estimation of irregular, realistic volumes, and intersections of volumes; there are no intermediate steps such as volume kriging of a multitude of discrete rectangular blocks and subsequent intersection of these with complex shapes and averaging of results.
- The methodology provides a direct measure of the reliability of estimation for any irregular volume, in terms of the associated standard error.
- The procedures are efficient in terms of computer processing and array storage requirements, to the extent that realtime interactive estimation of irregular volumes is now possible.

- The methodology offers full flexibility in terms of orientation of the estimation process to suit the geometry of the irregular volume and/or the anisotropy of the grade or quality which is estimated.
- The approach is flexible and provides a generic methodology for the application of virtually any estimation technique; i.e. it is not limited to the use of *ordinary* kriging.

PRACTICAL APPLICATION

The technology described herein has been incorporated in the LYNX Mining System and is available to the industry in a ready-to-use practical form for application to ore deposit evaluation, extraction simulation, and the requirements of operating mines (refer figure 8).

The system, and the new technology, provide a logical, practical, precise approach to modelling. The conceptual philosophy throughout is one of *what you see is what you get*. In other words, the precision and detail provided by the user, and displayed graphically on the screen, is that used by the system for analysis and estimation. There are no hidden, internal approximations.

BENEFITS OF THE NEW TECHNOLOGY

- The gross approximations and errors of most traditional modelling methods, when applied to complex deposits, have been eliminated.
- From the user's viewpoint, the whole approach to estimation is simpler, more logical and intuitively more correct in its application with the new technology. The interactive estimation capabilities encourage an improved appreciation of the three-dimensional geological and mining environment.
- The direct estimation of irregular volumes means that the "true" shapes of complex geological and mining units are accounted for in the modelling process.
- The ability to provide a more precise estimate, and a measure of the reliability of the estimate, allows management to relate the results to an acceptable degree of mining and financial risk.
- The interactiveness of the estimation procedures encourages the comparison of many design alternatives, and hence leads to optimization of the mining extraction process.
- Finally, the new technology provides a suitably precise platform for detailed planning and scheduling of mining operations.

CREDITS

3D Component Modelling Technology is the intellectual property and copyright of LYNX Geosystems Inc., Vancouver, Canada.

3D Geostatistics Technology is the intellectual property of Geostokos Ltd., London, U.K.

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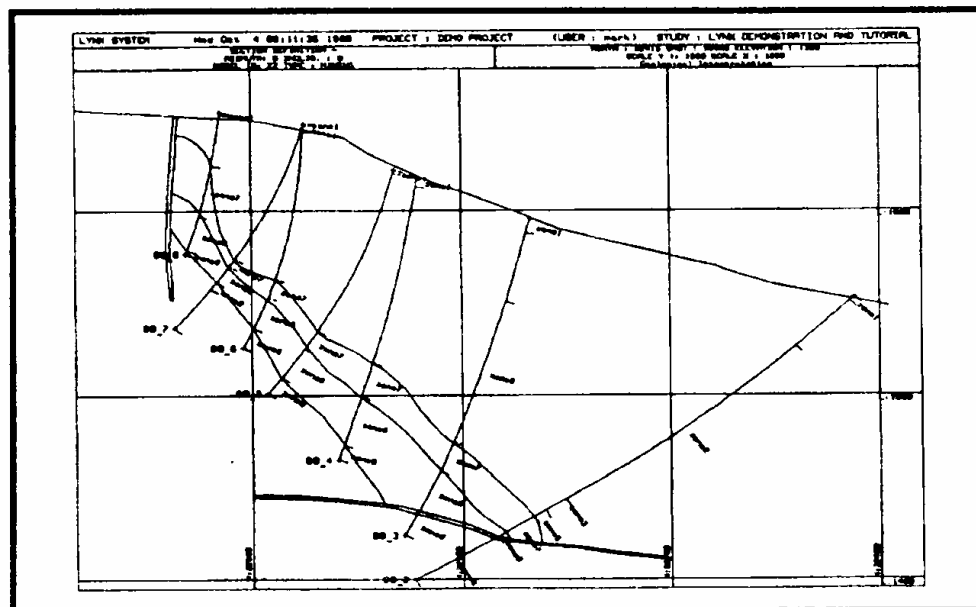


Figure 8: Practical Application of the New Modelling Technology
 (a) Geological Interpretation from Boreholes on Section

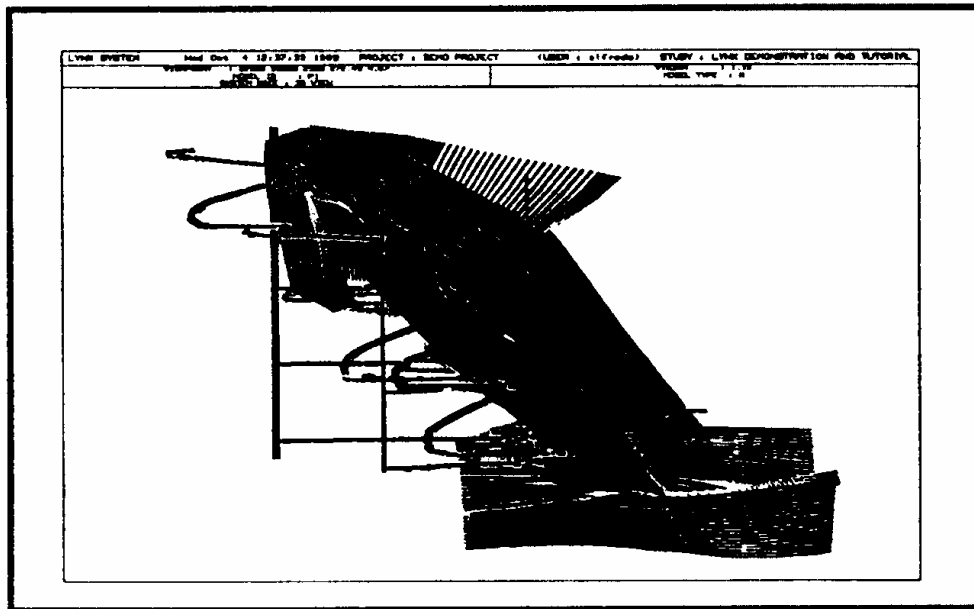


Figure 8: Practical Application of the New Modelling Technology
 (b) 3D Perspective of Deposit Ore Zones & Fault Structure

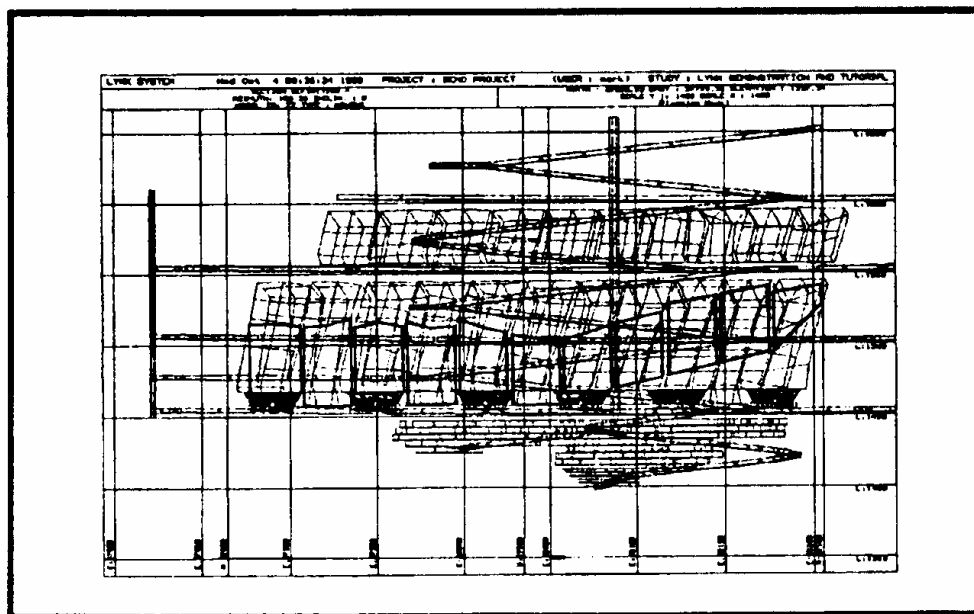


Figure 8: Practical Application of the New Modelling Technology
 (c) Design of Underground Mine Layout – Stopes & Mine Development on Section