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PRACTICAL KRIGING IN THREE DIMENSIONS

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Abstract Little has been published to date on the practical difficulties of ore reserve estimation in three-dimensional deposits. Some authors have suggested condensing the problem into two dimensions, but this is not always practicable or desirable. A suggestion and a FORTRAN IV Function Segment are provided which may alleviate some of these difficulties.

Key Words: FORTRAN, Geostatistics, Kriging, Regionalized variables, Semivariogram.

INTRODUCTION

At the feasibility study stage of a potentially mineable deposit, the ore-reserve valuation generally is carried out using data from (usually vertical) boreholes drilled through the deposit. Ideally for economic appraisal of the site, local as well as global estimates of ore grades and tonnages must be made. Where a deposit is of a seam or vein type, or where it comprises a sedimentary layer, the problem is essentially a two-dimensional one. The width (or depth or thickness) of the deposit, and the accumulation (grade times width) of metal may be considered as measurements or point samples in a two-dimensional plane (Sinclair and Deraisme, 1974). There may be some 'unrolling' problems, but essentially these are not statistical. However, in a three-dimensional deposit new problems occur. If local estimates are to be produced, these must be block estimates on a bench by bench basis. Many such applications are to be recognized, this author having experience with uranium, nickel, and lead/zinc deposits of disseminated character. In these, two-dimensional simplifications are not always possible. It is intended to use one of these deposits as an example throughout this paper. It is not possible to describe the deposit in full but for the purposes of this paper, it may be said that we have a disseminated nickel deposit in northern Norway, whose grades follow a spherical semivariogram with a range of influence of about 50 m. The deposit was explored by means of diamond drillholes—not always vertical. It was required to estimate the reserves on the basis of blocks 25 m by 25 m, with a 10 m bench height. This gave a total of approximately 4,000 blocks on 20 benches. A small deposit.

BLOCK ESTIMATION

To estimate each block by the method of Kriging, we must first set up the Kriging system of equations. These equations require that we must evaluate the average semivariogram between each of the samples we wish to include in the estimation procedure and the unknown area. Also we require the values of the average semivariogram between each pair of samples, and between all points within the unknown area. In one or two dimensions these evaluations present little or no problem.

In three, however, the problems can be great. Two approaches have been suggested, and this author ventures to suggest a third.

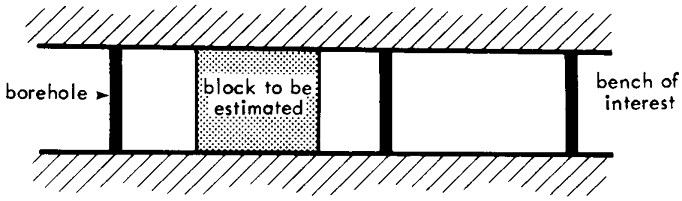

Figure 1. Considering deposit bench by bench.

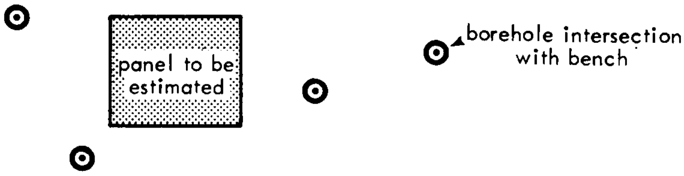

Figure 2. Bench reduced to two dimensions.

David (1976) suggests that a deposit may be considered bench by bench. That is, we ignore all data above and below the bench under consideration. Thus, both information and data span only the bench width (see Fig. 1) and the problem may be reduced to a two-dimensional one. Figure 2 shows the bench as if from above. The average grade of the borehole intersection then may be considered as coming from a point sample, and the block is reduced to a panel. This approach has many advantages as far as computation goes. Two-dimensional auxiliary functions (cf. Clark, 1976a) or relatively simple approximations may be used to evaluate the necessary semivariogram values, and hence the Kriging system. It is advantageous also in that only the average grade of the borehole intersections within each bench need be stored by the computer.

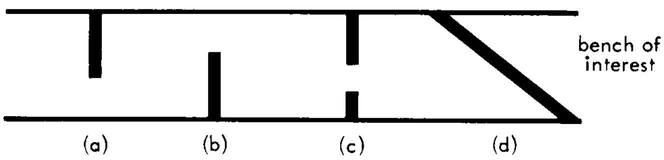

Figure 3. Problems with incomplete boreholes.

There are disadvantages to this approach, as illustrated in Figure 3. Boreholes may not make intersections with the full bench width. For example: boreholes may stop before the bottom of the bench (a); assaying may not have started until part way into the bench; (b) there may be core loss, or unassayed sections in the hole; (c) holes may be inclined, so that the intersection of the borehole with the bench is longer than the actual bench width. If these problems do not occur or it is considered an adequate approximation, then David's approach may be used. However, there is one other situation which cannot be handled by this approach. Suppose, as in our example, we have a bench width which is smaller than the range of influence of the semivariogram. Imagine a situation in which a borehole goes through, or close to a block to be estimated (Fig. 4). It is intuitively more reasonable to expect that the portions of borehole (i) *above and below* the block should have a closer relationship to, and hence more influence on, the shaded block than the portion of borehole (ii), say, on the same bench. That is, we would like to include sections of the boreholes above and below the bench in the Kriging system, because this should improve the estimation of the block grade considerably.

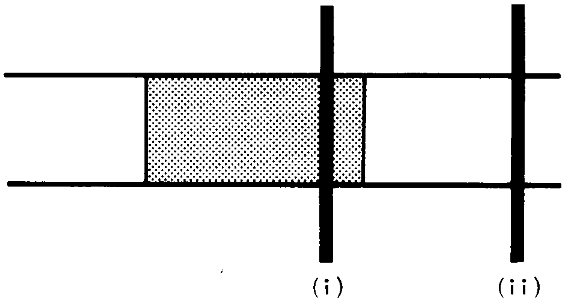

Figure 4. Considering core sections outside bench.

How much information above and below the block we wish to include in the estimation would depend on the bench width, size of the block, range of influence, and also on how the core is to be weighted in itself. Ideally, each core section which was assayed should be treated as a separate sample, and a Kriging system produced to determine the weight for each section. In that situation, obviously, the more sections included the lower the estimation variance. However, after a certain distance the increase in accuracy must become negligible-especially compared to the amount of work required to evaluate it. Alternatively, the core length could be split into the 'internal' (to the bench) portion, and the 'external' portions and each of these given an individual weight. This may become complicated, but might be worth doing if the range of influence is not greater than the bench size. A third possibility is to weight the whole length of core equally, that is average the length to be included in the estimation for each core and accord each one weight in the Kriging system. This approach is favored by the author in situations such as the Norwegian nickel example, because it forms a workable compromise between the ideal situation and the practicalities of computing in a reasonable size 3-D deposit. However, comparative studies remain to be carried out in any case study to ascertain the 'best' approach for the particular situation. Having decided upon this approach, it remains to evaluate the amount of core which yields the 'best' estimators. This could be determined by calculating the extension variance of a core of different length to a block of the desired size. However, this returns us to the problem of calculating semivariograms in three dimensions.

An alternative approach to David's is necessary once we have decided to work in three dimensions. Because 3-D auxiliary functions are intractable, we must produce numerical approximations to the required block-core, core-core, and block-block semivariograms. If cores are parallel and of the same length, core-core relationships can be evaluated by the 2-D auxiliary function $\gamma(\ell; b)$. Because our decision to work in 3-D was prompted partly by the fact that the cores are probably *not* of the same length, this simplification is unlikely to occur. The simplest numerical solution to calculating the average semivariograms between such pairs is to approximate each volume or length by a close network of points. A. Marechal (1975, pers. comm.) has stated that a block may be considered as a 4 x 4 x 4 network of points, and that this will produce an accuracy of 1 percent in the final semivariogram. Similarly a core might be considered to be a string of discrete points-presumably 64 points along the length would give similar accuracy. Then the average semivariogram between core and block would be calculated by evaluating the semivariogram between every point in the block and each point of the core (64^2 combinations) and then dividing by the number of combinations. Each average semivariogram would need 4096 calls to the point semivariogram in its calculation. This approach is heavy on computer time and (the author feels) uncertain of the final accuracy of both approximated semivariograms and final estimates.

A third approach has been suggested by this author elsewhere (Clark, 1976b). Briefly, it is suggested that a block be represented not by a network of points, but by an array of vertical 'cores'. That is, instead of $n \times n \times n$ points, a block is approximated by $n \times n$ core segments parallel to its vertical sides. Borehole cores are considered as themselves, not approximated at all. This reduces calculation of average semivariograms from n^6 point-point evaluations to n^4 core-core evaluations for the block-block average, n^2 for the block-core average, and 1 per core-core average. The block-block semivariogram is evaluated simply, because all the 'cores' are the same length and parallel to one another. The standard auxiliary function $\gamma(\ell; b)$ may be used here. Calculation of core-core or core-block semivariograms where one core is not parallel to another present a difficult problem - and are as yet unsolved. However, for boreholes which may be considered to be vertical (sic) the calculation of core-core relationships presents no problems at all. The author presents in Appendix I a FORTRAN IV Function Segment which will calculate the average semivariogram between any two cores of any lengths, in any relative positions and any distance apart so long as they are parallel, for a spherical model.

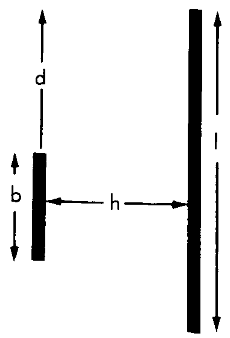

Figure 5. Average semivariogram GIMEL-typical situation.

The function is called GIMEL (the Hebrew letter G) and is used as follows (see Fig. 5):

GAM = GIMEL (EL, B, H, D, A, C),

where A is the range of influence and C the sill of the point semivariogram (spherical type); GAM is to contain the average semivariogram between two parallel cores of length ℓ (EL) and b (B), a distance h (H) apart, whose ends are offset by a length d (D) as illustrated. Restrictions on the parameters are: ℓ and b must be positive, b must not be greater than ℓ. If the user finds this restrictive, a simple insertion of about six lines will generalize the FUNCTION. d may take any value. h may be positive or zero. This last enables different lengths on one borehole to be compared.

OTHER USES OF GIMEL

Although the use of the core-core function reduces the calculation of bench/block estimations considerably in the regular situation, this is not the only situation in which it might be useful. Approximating one of the three dimensions by a continuous line allows greater attention to be paid to irregularities which may occur. For example, in Eire, Pb/Zn deposits generally are confined to one stratum of the limestone. The relatively irregular boundaries of this stratum make the estimation of block averages inaccurate if not meaningless. However, if the boundary between the strata can be fairly well defined (as is usual) then estimation of an irregular volume is relatively simple with the core-core approach. Figure 6 illustrates how a block or stope near the irregular hanging wall might be approximated by cores, so that GIMEL then could be used for the average semivariograms. Similarly other irregularly shaped stopes, or edge blocks in an open pit could be estimated with no problem.

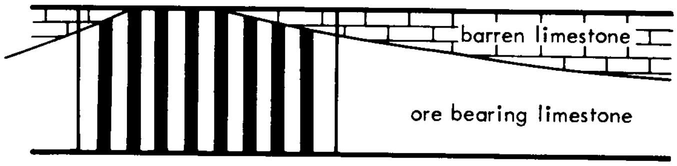

Figure 6. Irregular blocks caused by strata horizons.

AN EXAMPLE

Producing simple tables for checking GIMEL is difficult because of the four arguments. However, a program example is given in Appendix II with its output, to enable users to check the FUNCTION. Some checks also may be made against the standard function $\gamma(\ell; b)$ by computing GIMEL *(b, b,* ℓ*, 0, a, c)*. Also GIMEL (ℓ, ℓ, *0, h, a,* c) may be compared with regularized semivariogram for length ℓ.

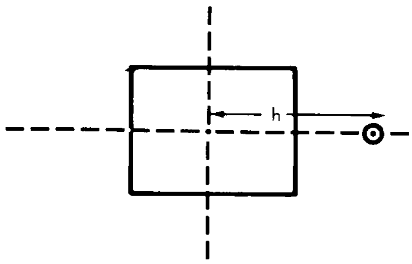

Figure 7. Borehole value is being extended to block.

The example used is a program to determine the extension variance of a vertical core section of length ℓ, at a distance *h* from the center of a block along a median through the block (see Fig. 7). The disseminated nickel example has been used, that is a = *50, c* = 1.0, blocks are 25 m by 25 m, benches are 10 m. A grid of 8 by 8 'cores' has been used to approximate the block. Values of *h* range from zero, sample central to block, to 50 m, that is sample almost out of range. Lengths of the core range from 2 m to 50 m and are centered on the center of the bench. Also it may be noticed, the program takes advantage of any symmetry which might be present.

Acknowledgments - The function GIMEL was first conceived and initially evaluated while the author was employed by the Norwegian Geological Survey in 1974. The programs described in this paper were produced when the author was Visiting Professor of Geology at Syracuse University, Syracuse, New York and-were implemented on their DEC System-10 computer.

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